



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

TECHNICAL MATHEMATICS P1

2021

MARKS: 150

TIME: 3 hours

**This question paper consists of 11 pages, a 2-page information sheet
and 1 answer sheet.**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTION 4.2.2 on the ANSWER SHEET provided. Write your centre number and examination number in the space provided on the ANSWER SHEET and hand in the ANSWER SHEET with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x :

1.1.1 $(3 - x)(x + 1) = 0$ (2)

1.1.2 $2x^2 = 3x + 7$ (correct to TWO decimal places) (4)

1.1.3 $x(x - 5) \leq 0$ (2)

1.2 Solve for x and y if:

$y + x = 3$ and $x^2 + y^2 = 89$ (6)

1.3 The formula for calculating the electrical force between two charges is given by:

$$F = \frac{K Q_1 Q_2}{r^2} \text{ where}$$

F = the force in N

K = Coulomb's constant in Nm^2/C^2

r = distance in m

Q_1 = charge in C

Q_2 = charge in C

1.3.1 Make r the subject of the formula. (2)

1.3.2 Hence, or otherwise, calculate the distance r if:

$$F = 2,25 \times 10^{-4} \text{ N}$$

$$K = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$Q_1 = 0,5 \times 10^{-6} \text{ C}$$

$$Q_2 = 0,2 \times 10^{-6} \text{ C}$$

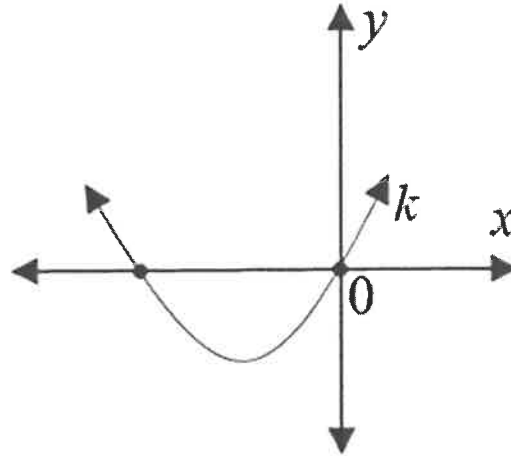
(2)

1.4 Evaluate: $1101_2 + 111_2$ (Leave your answer in binary form.) (2)

[20]

QUESTION 2

2.1 The graph below represents function k defined by $k(x) = x^2 + 3x$.



Describe the nature of the real roots of k . (2)

2.2 Show that the roots of $x^2 + px - 2p^2 = 0$ are rational for $p \in$ real numbers. (4)
[6]

QUESTION 3

3.1 Simplify (showing ALL calculations) the following WITHOUT using a calculator:

3.1.1 $\sqrt{16 a^6}$ (2)

3.1.2 $\sqrt{\log_2 32 + \log 100 + 9}$ (3)

3.1.3 $(4\sqrt{5} + \sqrt{2})(\sqrt{2} - 4\sqrt{5})$ (2)

3.2 Solve for x : $\log_3 x = 3 - \log_3 (x+6)$ (6)

3.3 Given the complex number $z = 2w - 7i$ where $w = \frac{1}{2} + 3i$

3.3.1 Determine z in the form $a + bi$ (2)

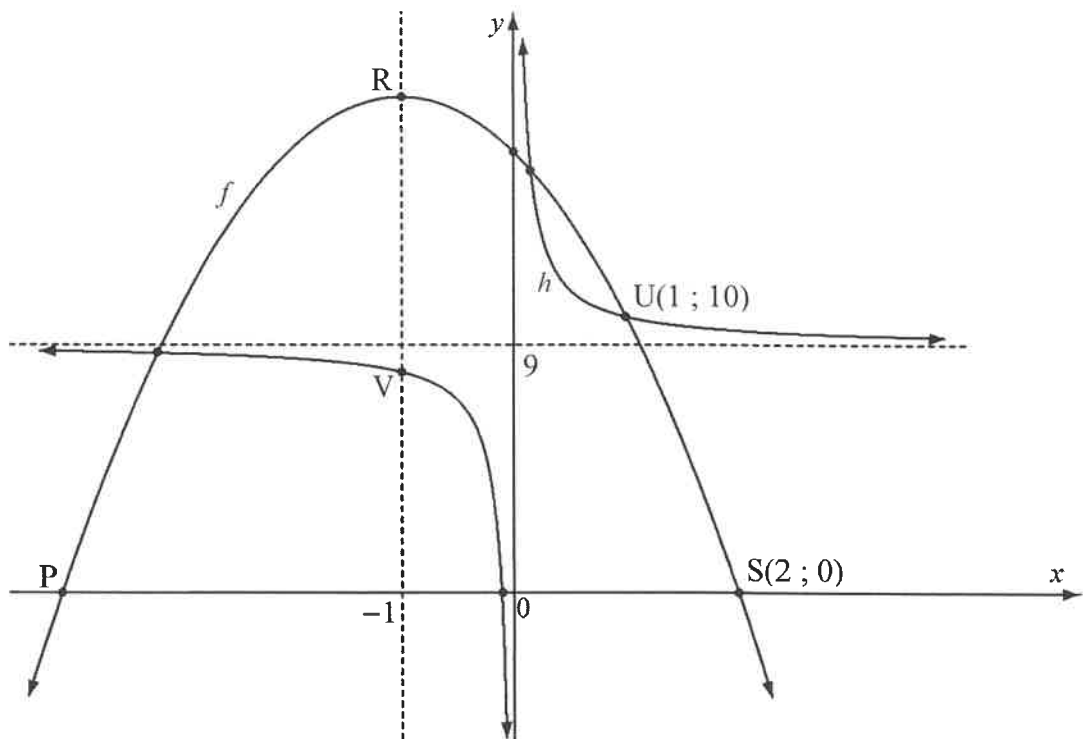
3.3.2 Express z in the polar form $z = r \text{ cis } \theta$ (where θ is in degrees) (5)

3.4 Solve for a and b if $a + b + ai - bi = 5 - 3i$ (4)

[24]

QUESTION 4

- 4.1 Sketched below are the graphs of functions defined by $f(x) = ax^2 + bx + c$ and $h(x) = \frac{k}{x} + q$ with $U(1; 10)$ one of the points of intersection of f and h .
 The equation of the asymptote of h is $y = 9$.
 $x = -1$ is the equation of the axis of symmetry of f .
 P and $S(2; 0)$ are the x -intercepts of f .
 R is the turning point of f .
 V is a point on both h and the axis of symmetry of f .



- 4.1.1 Write down the domain of h . (1)
- 4.1.2 Write down the coordinates of P . (1)
- 4.1.3 Determine:
- (a) The equation of f (4)
 - (b) The equation of h (3)
- 4.1.4 Determine the length of RV . (3)
- 4.1.5 For which value(s) of x is $\frac{h(x)}{f(x)}$ undefined? (3)

4.2 Given the functions defined by $g(x) = (1,495)^x - 5$ and $p(x) = -\sqrt{16 - x^2}$

4.2.1 Determine:

(a) The y -intercept of g (1)

(b) The x -intercept(s) of g (correct to the nearest integer) (3)

4.2.2 Sketch the graphs of g and p on the same set of axes on the ANSWER SHEET provided. Clearly show the intercepts with the axes and any asymptote(s). (5)

4.2.3 Use your sketch graph to:

(a) Write down the range of p (2)

(b) Determine the equation of a line passing through the intercepts of g in the form $y = \dots$ (2)

(c) Determine the values(s) of x for which $g(x) > p(x)$ (2)

(d) Write down the values(s) of x , where $p'(x) \cdot g(x) \geq 0$ (2)

[32]

QUESTION 5

- 5.1 The annual effective interest rate charged is 8,5%. Calculate the nominal interest rate charged per annum if it is compounded quarterly. (3)
- 5.2 The value of a diesel generator used to supply electricity to a hospital depreciated to an amount of R152 523 at the end of 3 years. Use the reducing-balance method and determine the initial value of the generator if it depreciated at a rate of 11% per annum. (3)
- 5.3 Nosizwe and Martin are co-owners of a mechanical workshop. They plan to upgrade the workshop within 5 years and will require an amount of R35 000 for the upgrade. They each make separate investments at a financial institution as follows:

Nosizwe:	<ul style="list-style-type: none"> • Invests R8 000 at a rate of 7,64% per annum, compounded monthly. • At the end of 2 years, the interest rate changed to 8,12% per annum, compounded annually. • At the beginning of the 3rd year, she further invested an amount of R5 000.
Martin:	<ul style="list-style-type: none"> • Invests R13 000 in an account that earns 5,8% per annum simple interest.

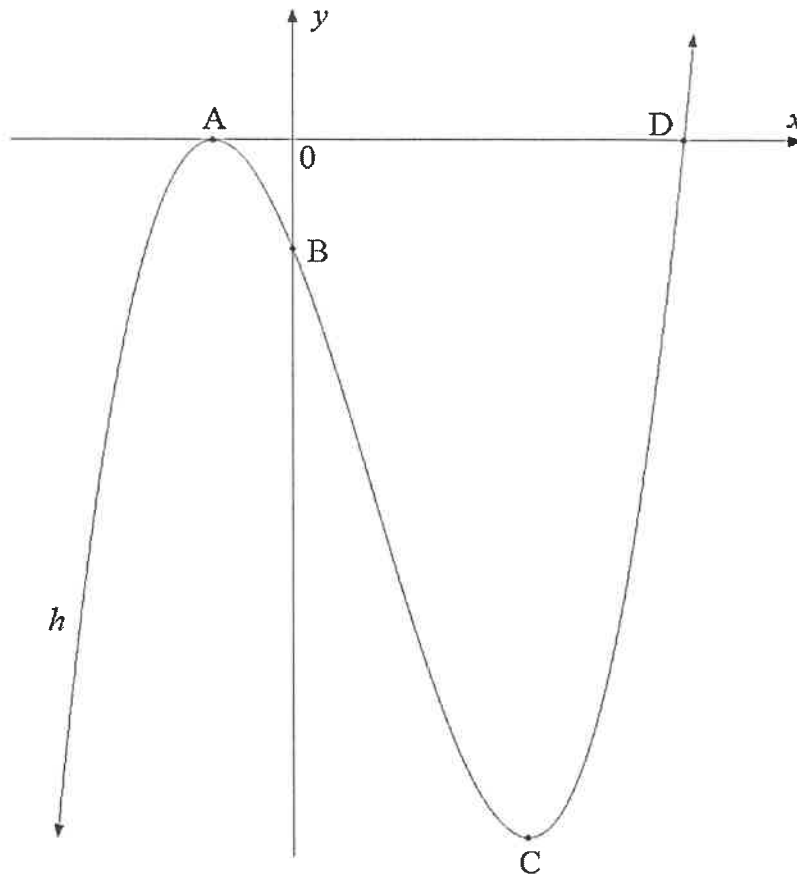
- 5.3.1 Determine the amount of money that Martin will receive at the end of the 5-year investment period. (2)
- 5.3.2 Hence, determine whether they will have jointly accumulated enough money for the upgrades at the end of the 5-year investment period. (5)
- [13]**

QUESTION 6

- 6.1 Determine $f'(x)$ using FIRST PRINCIPLES if $f(x) = 2x + 3$ (5)
- 6.2 Determine:
- 6.2.1 $\frac{dy}{dx}$ if $y = -x^{-5} + 3x^4$ (2)
- 6.2.2 $f'(x)$ if $f(x) = \frac{3}{x^4} - \frac{x}{\sqrt{x}}$ (4)
- 6.2.3 $D_x \left[\frac{x^2 + x - 6}{x + 3} \right]$ (3)
- 6.3 Determine the average gradient of the function defined by $h(x) = -2x^2 + 2$ between the points where $x = 0$ and $x = 2$ (3)
- 6.4 Given: $g(x) = 1 - x^2$
- 6.4.1 Determine the gradient of the tangent to g at the point where $x = -3$ (2)
- 6.4.2 Hence, determine the equation of a tangent to g at the point where $x = -3$ (3)
- [22]

QUESTION 7

- 7.1 The graph below represents the function defined by $h(x) = x^3 - 3x^2 - 9x - 5$
 A and C are the turning points of h .
 A, B and D are intercepts on the axes.



- 7.1.1 Write down the coordinates of B. (1)
- 7.1.2 Show that $x + 1$ is a factor of h . (2)
- 7.1.3 Hence, determine the coordinates of D. (3)
- 7.1.4 Determine the coordinates of C. (5)
- 7.2 Write down the values of x for which h is increasing. (2)
- [13]**

QUESTION 8

The displacement (distance), s , in metres, travelled by car A over time (t) in seconds, after the brakes were applied until the car stopped, is represented by a formula $s = 30t - 3t^2$

8.1 If $\frac{ds}{dt} = 0$, determine the time t (in seconds).

HINT: $\frac{ds}{dt}$ is the velocity of the car in m/s. (2)

8.2 Determine:

8.2.1 The velocity, in kilometres per hour, at the time when the brakes were first applied (3)

8.2.2 The maximum distance travelled by car A before it stopped (2)

8.3 State, with reasons, whether car A will collide with stationary car B, which is 70 m directly in front of car A, after the brakes have been applied. (2)

[9]

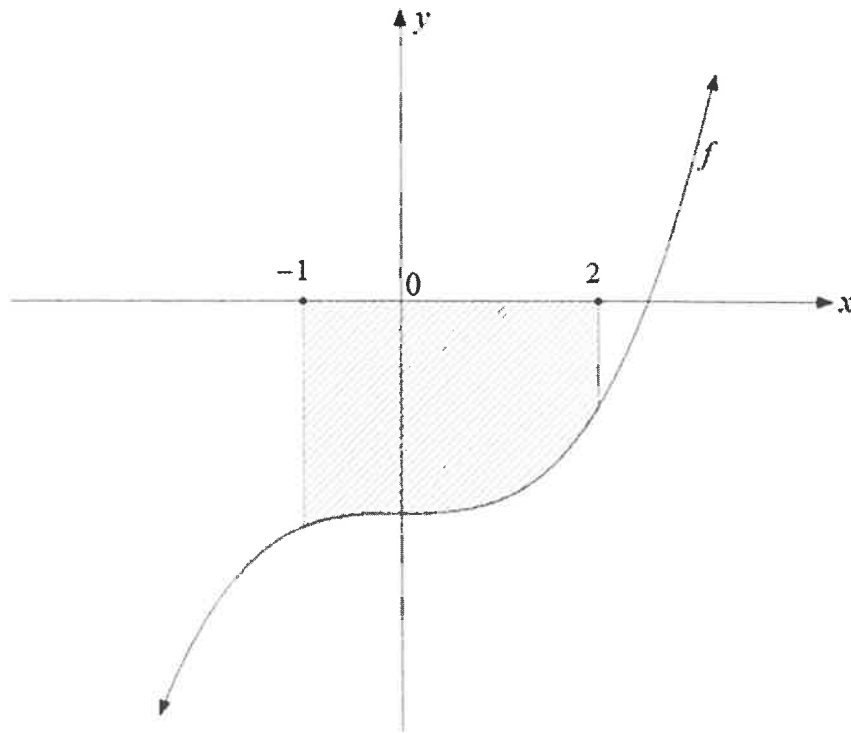
QUESTION 9

9.1 Determine the following integrals:

9.1.1 $\int \left(x^{-2} + \frac{1}{x} \right) dx$ (3)

9.1.2 $\int \left(x^{\frac{1}{3}} - 5x^4 \right) dx$ (2)

9.2 The sketch below represents the shaded bounded area of the curve of the function defined by $f(x) = 2x^3 - 4$



Determine (showing ALL calculations) the shaded area bounded by the curve and the x-axis between the points where $x = -1$ and $x = 2$

(6)
[11]

TOTAL: 150



INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a}$$

$$y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + in)$$

$$A = P(1 - in)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$i_{\text{eff}} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \quad x > 0$$

$$\int a^x dx = \frac{a^x}{\ln a} + C, \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{Area of } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2 + \cos^2 = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \text{cosec}^2 \theta$$

$$\pi \text{ rad} = 180^\circ$$

Angular velocity $= \omega = 2\pi n = 360^\circ n$ where $n =$ rotation frequency

Circumferential velocity $= v = \pi Dn$ where $D =$ diameter and $n =$ rotation frequency

$s = r\theta$ where $r =$ radius and $\theta =$ central angle in radians

Area of a sector $= \frac{rs}{2} = \frac{r^2\theta}{2}$ where $r =$ radius, $s =$ arc length and
 $\theta =$ central angle in radians

$4h^2 - 4dh + x^2 = 0$ where $h =$ height of segment, $d =$ diameter of circle and
 $x =$ length of chord

$A_T = a(m_1 + m_2 + m_3 + \dots + m_n)$ where $a =$ equal parts, $m_1 = \frac{o_1 + o_2}{2}$
 and $n =$ number of ordinates

OR

$A_T = a \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1} \right)$ where $a =$ equal parts, $o_i = i^{\text{th}}$ ordinate
 and $n =$ number of ordinates

