basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 16 pages.
Hierdie nasienriglyne bestaan uit 16 bladsye.

Copyright reserved/*Kopiereg voorbehou*

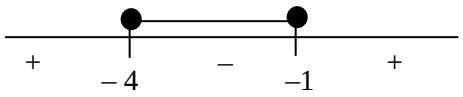
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in all aspects of the marking guidelines.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, sien slegs die EERSTE poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyne van toepassing.*

QUESTION/VRAAG 1

1.1.1	$x^2-2x-24=0$ $(x-6)(x+4)=0$ $x=6$ or $x=-4$	✓factors ✓$x=6$ ✓$x=-4$ (3)
1.1.2	$2x^2-3x-3=0$ $x=\dfrac{3\pm\sqrt{(-3)^2-4(2)(-3)}}{2(2)}$ $x=\dfrac{3\pm\sqrt{33}}{4}$ $x=2{,}19$ or $x=-0{,}69$	✓substitution into the correct formula ✓$x=2{,}19$ ✓$x=-0{,}69$ (3)
1.1.3	$x^2+5x\le -4$ $x^2+5x+4\le 0$ $(x+4)(x+1)\le 0$ Critical values: $x=-4$ or $x=-1$  $-4\le x\le -1$ **OR/*OF*** $x\in[-4\,;-1]$	✓ standard form ✓ critical values ✓✓answer (4)
1.1.4	$\sqrt{x+28}=2-x$ $\left(\sqrt{x+28}\right)^2=(2-x)^2$ $x+28=4-4x+x^2$ $x^2-5x-24=0$ $(x-8)(x+3)=0$ $x\ne 8$ or $x=-3$	✓ squaring both sides ✓ standard form ✓ factors ✓ answers with selection (4)

1.2	$2y=3+x$ $x=2y-3$ **...** (1) $2xy+7=x^2+4y^2$ ... (2) $2y(2y-3)+7=(2y-3)^2+4y^2$ $4y^2-6y+7=4y^2-12y+9+4y^2$ $4y^2-6y+2=0$ $2y^2-3y+1=0$ $(2y-1)(y-1)=0$ $y=\frac{1}{2}$ or $y=1$ $x=-2$ or $x=-1$ **OR/*OF*** $2y=3+x$ $y=\frac{3}{2}+\frac{x}{2}$ ...(1) $2xy+7=x^2+4y^2$ ... (2) $2x\left(\frac{3}{2}+\frac{x}{2}\right)+7=x^2+4\left(\frac{3}{2}+\frac{x}{2}\right)^2$ $3x+x^2+7=x^2+9+6x+x^2$ $x^2+3x+2=0$ $(x+2)(x+1)=0$ $x=-2$ or $x=-1$ $y=\frac{1}{2}$ or $y=1$	✓equation 1 ✓substitution ✓simplification ✓standard form ✓y – values ✓x – values (6) **OR/*OF*** ✓equation 1 ✓substitution ✓simplification ✓standard form ✓x – values ✓y – values (6)
1.3	$\frac{n}{m}=\frac{p}{n}$ $n^2=mp$ $\Delta=b^2-4ac$ $\Delta=n^2-4mp$, but $n^2=mp$ $\Delta=n^2-4n^2$ **OR/*OF*** $\Delta=mp-4mp$ $\Delta=-3n^2$ $\Delta=-3mp$ $n^2>0$ $mp>0$ $\therefore -3n^2<0$ $\therefore -3mp<0$ $\therefore \Delta<0 \Rightarrow x$ is a non-real number	✓$\frac{n}{m}=\frac{p}{n}$ ✓$n^2=mp$ ✓$\Delta=-3n^2$ or $-3mp$ ✓$\Delta<0$ (4)
		[24]

QUESTION/VRAAG 2

2.1	$\frac{90}{x}=\frac{81}{90}$ $81x=8\,100$ $x=100$ **OR/OF** $x=90\times\frac{10}{9}$ $x = 100$	✓ $\frac{90}{x}=\frac{81}{90}$ ✓ answer (2) **OR/OF** ✓ $\frac{10}{9}$ ✓ answer (2)
2.2	$S_n=\frac{a\left(1-r^n\right)}{1-r}$ $S_n=\frac{100\left(1-(0{,}9)^n\right)}{1-0{,}9}$ $S_n=\frac{100\left(1-(0{,}9)^n\right)}{0{,}1}$ $\therefore S_n=1\,000\left(1-(0{,}9)^n\right)$	✓r = 0,9 ✓substitution into correct formula (2)
2.3	$S_\infty=\frac{a}{1-r}$ $S_\infty=\frac{100}{1-\frac{9}{10}}$ $S_\infty=1000$ **OR/OF** $S_\infty=\lim_{n\to\infty}\left[1\,000\left(1-(0{,}9)^n\right)\right]$ $S_\infty=1000$	✓ substitution ✓ answer (2) **OR/OF** ✓ $S_\infty=\lim_{n\to\infty}\left[1\,000\left(1-(0{,}9)^n\right)\right]$ ✓ answer (2)
		[6]

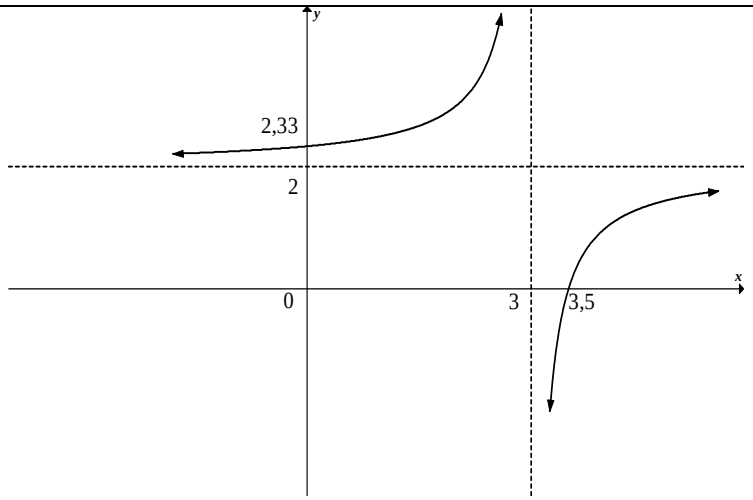
QUESTION 3

3.1	–82	✓ answer (1)
3.2	–145 ; –122 ; –101 ; . . . 23 21 –2 –2 $2a=-2 \quad \therefore\ a=-1$ $3a+b=23 \quad \therefore 3(-1)+b=23 \quad \therefore b=26$ $a+b+c=-145 \ \therefore -1+26+c=-145 \ \therefore\ c=-170$ $\therefore T_n=-n^2+26n-170$ **OR/OF** $2a=-2 \quad \therefore\ a=-1$ $c=-145+(-2)-23=-170$ $\therefore T_n=-n^2+bn-170$ $-145=-1+b-170$ $b=26$ $\therefore T_n=-n^2+26n-170$	✓ $2a=-2$ ✓ $3(-1)+b=23$ ✓ $-1+26+c=-145$ (3) **OR/OF** ✓ $2a=-2$ ✓ $c=-145+(-2)-23$ ✓ $-145=-1+b-170$ (3)
3.3	$T_n=bn+c$ or $T_n=a+(n-1)d$ $T_n=-2n+25$ $=23+(n-1)(-2)$ $=25-2n$ $-2n+25=-121$ $-2n=-146$ $n=73$ Between T_{73} and T_{74} **OR/*OF*** $T_{n+1}-T_n=-(n+1)^2+26(n+1)-170-(-n^2+26n-170)$ $-121=-2n+25$ $n=73$ Between T_{73} and T_{74}	✓ $T_n=-2n+25$ ✓ $T_n=-121$ ✓ $n=73$ ✓answer (4) **OR/*OF*** ✓ $T_n=-2n+25$ ✓ $T_n=-121$ ✓ $n=73$ ✓ answer (4)
3.4	$n=\frac{-b}{2a}=\frac{-26}{2(1)}=13$ $T_{13}=-1$ $\therefore$ add 2 **OR/*OF*** $T'_n=-2n+26=0$ $n=13$ $T_{13}=-(13)^2+26(13)-170=-1$ $\therefore$ add 2	✓13 ✓ $T_{13}=-1$ ✓ add 2 (3) **OR/*OF*** ✓13 ✓ $T_{13}=-1$ ✓ add 2 (3)
		[11]

QUESTION/*VRAAG* 4

4.1	$a = 5$ and/*en* $d = 2$ $T_{51} = 5 + (51-1)(2)$ $= 105$	✓ a and d ✓ substitution into correct formula ✓ answer (3)
4.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{51} = \frac{51}{2}[2(5) + (51-1)2]$ or/*of* $S_{51} = \frac{51}{2}[5 + 105]$ $= 2\ 805$ $= 2\ 805$	✓ substitution into correct formula ✓ answer (2)
4.3	$\sum_{n=1}^{5\,000}(2n+3) = 5 + 7 + 9 + ... + 10\ 003$	✓ expansion (1)
4.4	$T_1 = -3$ $T_{4\,999} = -2(4\,999) - 1 = -9\,999$ $\therefore \sum_{n=1}^{5000}(2n+3) + \sum_{n=1}^{4999}(-2n-1)$ = (5 + 7 + 9 + … + 9 999 + 10 001 + 10 003) + (–3 – 5 – 7 – 9 – … – 9 999) = 10 001 + 10 003 –3 = 20 001 **OR/OF** $S_{4\,999} = \frac{4\,999}{2}[2(-3) + (4\,999-1)(-2)] = -24\,999\,999$ $S_{5\,000} = \frac{5000}{2}((2)(5) + (5\,000-1)(2)) = 25\,020\,000$ $\sum_{n=1}^{5\,000}(2n+3) + \sum_{n=1}^{4\,999}(-2n-1) = 25\,020\,000 - 24\,999\,999$ $= 20\,001$	✓ $T_1 = -3$ ✓ $T_{4999} = -9\,999$ ✓both expansions ✓ answer (A) (4) **OR/OF** ✓ $T_1 = -3$ ✓ $S_{4\,999} = -24\,999\,999$ ✓ $S_{5\,000} = 25\,020\,000$ ✓ answer (A) (4)
		[10]

QUESTION/VRAAG 5

5.1	$x = 3$ $y = 2$	✓ $x = 3$ ✓ $y = 2$ (2)
5.2	$x \in R,\ x \neq 3$ **OR/*OF*** $x \in (-\infty\ ;3) \cup (3\ ;\infty)$ **OR/OF** $x < 3$ or $x > 3$	✓answer (1) **OR/OF** ✓answer (1) **OR/OF** ✓answer (1)
5.3	$0 = \frac{-1}{x-3} + 2$ $-2x + 6 = -1$ $x = \frac{7}{2}$ *x*-int: $\left(\frac{7}{2}\ ;\ 0\right)$	✓ $y = 0$ ✓ answer (2)
5.4	*y*-int: $\left(0\ ;\ \frac{7}{3}\right)$	✓ $x = 0$ ✓ $\frac{7}{3}$ (2)
5.5		✓ asymptotes ✓ intercepts with the axes ✓ shape (3)
		[10]

QUESTION/VRAAG 6

6.1	$f(x)=\log_4 x$ $2=\log_4 k$ $4^2=k$ $\therefore k=16$	✓ substitution of (k ; 2) ✓ answer (2)
6.2	$-1=\log_4 x \quad \therefore \ x=\frac{1}{4}$ $\frac{1}{4}\le x\le 16$ or/of $x\in\left[\frac{1}{4}\ ;\ 16\right]$	✓ $x=\frac{1}{4}$ ✓ answer (2)
6.3	$f(x)=\log_4 x$ $y=\log_4 x$ $x=\log_4 y$ $y=4^x$	✓swopping x and y ✓answer (2)
6.4	$x<0$ **OR/*OF*** $x\in(-\infty\ ;\ 0)$	✓✓answer (2) **OR/*OF*** ✓✓answer (2)
		[8]

QUESTION 7

7.1	B(–4 ; 0) D(6 ; 0)	✓ B(–4 ; 0) ✓ D(6 ; 0) (2)
7.2	$f(x)=x^2-2x-24$ $x_{tp}=\frac{-b}{2a}$ **OR/*OF*** $2x-2=0$ **OR/OF** $x=\frac{-4+6}{2}$ $x=\frac{-(-2)}{2(1)}$ $\therefore\ x_{tp}=1$ $y_{tp}=f(1)$ $=1^2-2(1)-24$ $=-25$ C(1 ; –25)	✓ $x_{tp}=1$ ✓ $y_{tp}=-25$ (2)
7.3	$y\geq -25$ **OR/*OF*** $y\in[-25\ ;\ \infty)$	✓ answer (1) **OR/OF** ✓ answer (1)
7.4.1	$m_{AE}=\tan 14,04°=0,25=\frac{1}{4}$	✓ answer (1)
7.4.2	$m_{\text{tang}}=-4$ $f'(x)=2x-2$ $2x-2=-4$ $x_T=-1$ $y_T=-21$	✓ $m_{\text{tang}}=-4$ ✓ $f'(x)=2x-2$ ✓ equating ✓ $x_T=-1$ ✓ $y_T=-21$ (5)
7.5	$m_{line}=\frac{1}{4}$ $y+9=\frac{1}{4}(x+3)$ **OR/OF** $-9=\frac{1}{4}(-3)+c$ $y+9=\frac{1}{4}x+\frac{3}{4}$ $c=-\frac{33}{4}=-8,25$ $y=\frac{1}{4}x-\frac{33}{4}$ **OR/OF** $y=0,25x-8,25$ $x^2-2x-24=\frac{1}{4}x-\frac{33}{4}$ $4x^2-8x-96=x-33$ $4x^2-9x-63=0$ $(4x-21)(x+3)=0$ $\therefore x=\frac{21}{4}=5,25$ or $x\neq -3$	✓ $m_{line}=\frac{1}{4}$ ✓ substitution *m* and K(– 3 ; – 9) ✓ $y=\frac{1}{4}x-\frac{33}{4}$ ✓ equating ✓ standard form ✓ answer with selection (6)
		[17]

QUESTION/VRAAG 8

8.1	$A = P(1-i)^n$ $A = 980\ 000(1-0,092)^7$ $A = R498\ 685,82$	✓ correct formula ✓ substitution ✓ answer (A) (3)
8.2	$A = P(1+i)^n$ $116\ 253,50 = 75\,000\left(1+\frac{0,068}{4}\right)^{4n}$ $1,550\ 046\ 667 = (1,017)^{4n}$ $\log(1,550\ 046\ 667) = 4n\log(1,017)$ $4n = \frac{\log(1,550\ 046\ 667)}{\log(1,017)}$ or $4n = \log_{1,017}(1,550\ 046\ 667)$ $4n = 25,99\ \ldots$ $n =$ 6,50 years	✓ $\frac{0,068}{4}$ ✓ substitution in correct formula ✓ correct use of logs ✓ answer (4)
8.3.1	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $450\ 000 = \frac{x\left[\left(1+\frac{0,0835}{12}\right)^{60} - 1\right]}{\frac{0,0835}{12}}$ $x = R6\ 068,69$	✓ $i = \frac{0,0835}{12}$ ✓ substitution into correct formula ✓ answer (3)
8.3.2(a)	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $P = \frac{11\ 058,85\left[1-\left(1+\frac{0,12}{12}\right)^{-4\times12}\right]}{\frac{0,12}{12}}$ $P = R419\ 948,32$ **OR/*OF*** $\text{Balance} = A - F$ $= P(1+i)^n - \frac{x\left[(1+i)^n - 1\right]}{i}$ $=1\ 050\ 000\left(1+\frac{0,12}{12}\right)^{12\times21} - \frac{11\ 058,85\left[\left(1+\frac{0,12}{12}\right)^{12\times21} - 1\right]}{\frac{0,12}{12}}$ $=R12\ 887\ 702,20 - R12\ 467\ 749,81$ $=R419\ 952,39$	✓ $n = 48$ in P-formula ✓ substitution into correct formula ✓ answer (A) (3) **OR/*OF*** ✓ $n = 252$ in both formulae ✓ subst into correct formulae ✓ answer (A) (3)

8.3.2(b)	Total paid $= 11\,058,85 \times 21 \times 12 = 2\,786\,830,20$ Loan Paid = $1\,050\,000 - 419\,948,32 = 630\,051,68$ Interest paid = 2 786 830,20 – 630 051,68 = R2 156 778,52 **OR/OF** Total paid $= 11\,058,85 \times 21 \times 12 = 2\,786\,830,20$ Loan Paid = $1\,050\,000 - 419\,952,39 = 630\,047,61$ Interest paid = 2 786 830,20 – 630 047,61 = R2 156 782,59 **OR/OF** Interest paid = 11 058,85 × 21 × 12 – (1 050 000 – 419 948,32) = 2 786 830,20 – 630 051,68 = R2 156 778,52	✓ $11\,058,85 \times 21 \times 12$ ✓ 1 050 000 – Balance Outstanding ✓ answer (3) **OR/OF** ✓ $11\,058,85 \times 21 \times 12$ ✓ 1 050 000 – Balance Outstanding ✓ answer (3) **OR/OF** ✓ $11\,058,85 \times 21 \times 12$ ✓ 1 050 000 – Balance Outstanding ✓ answer (3)
		[16]

QUESTION/VRAAG 9

9.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{2x^2+4xh+2h^2-3x-3h-2x^2+3x}{h}$ $=\lim\limits_{h\to 0}\dfrac{4xh+2h^2-3h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(4x+2h-3)}{h}$ $=\lim\limits_{h\to 0}(4x+2h-3)$ $\therefore f'(x)=4x-3$ **OR/OF** $f(x)=2x^2-3x$ $f(x+h)=2(x+h)^2-3(x+h)$ $f(x+h)=2x^2+4xh+2h^2-3x-3h$ $f(x+h)-f(x)=4xh+2h^2-3h$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{4xh+2h^2-3h}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(4x+2h-3)}{h}$ $=\lim\limits_{h\to 0}(4x+2h-3)$ $\therefore f'(x)=4x-3$	✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5) **OR/OF** ✓substitution ✓ $2x^2+4xh+2h^2-3x-3h$ ✓ $4xh+2h^2-3h$ ✓factorisation ✓answer (5)
9.2.1	$y=4x^5-6x^4+3x$ $\dfrac{dy}{dx}=20x^4-24x^3+3$	✓ $20x^4$ ✓ $-24x^3$ ✓3 (3)

9.2.2	$D_x\left[\frac{-\sqrt[3]{x}}{2}+\left(\frac{1}{3x}\right)^2\right]$	
	$D_x\left[\frac{-x^{\frac{1}{3}}}{2}+\frac{x^{-2}}{9}\right]$	✓$\frac{-x^{\frac{1}{3}}}{2}$ ✓$\frac{x^{-2}}{9}$
	$D_x\left[-\frac{1}{2}x^{\frac{1}{3}}+\frac{1}{9}x^{-2}\right]$	
	$=-\frac{1}{6}x^{-\frac{2}{3}}-\frac{2x^{-3}}{9}$	✓$-\frac{1}{6}x^{-\frac{2}{3}}$ ✓$-\frac{2x^{-3}}{9}$
	$=-\frac{1}{6x^{\frac{2}{3}}}-\frac{2}{9x^3}$	(4)
		[12]

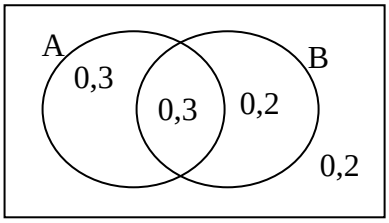
QUESTION/VRAAG 10

10.1	$h(x) = ax^3 + bx^2$ $h'(x) = 3ax^2 + 2bx$ $h'(4) = 0$ $48a + 8b = 0$ $6a + b = 0$ ...(1) $h(4) = 32$ $64a + 16b = 32$ $4a + b = 2$ ...(2) (1)– (2): $6a + b = 0$ $4a + b = 2$ $2a = -2$ $a = -1$ $4(-1) + b = 2$ $b = 6$	✓ $h'(x)$ ✓ $h'(4) = 0$ ✓ $48a + 8b = 0$ or $6a + b = 0$ ✓ $h(4) = 32$ ✓ $64a + 16b = 32$ or $4a + b = 2$ (5)
10.2	$h(x) = -x^3 + 6x^2$ $-x^3 + 6x^2 = 0$ $x^2(-x + 6) = 0$ $x = 0$ or $x = 6$ $\therefore$ A(6 ; 0)	✓ $-x^3 + 6x^2 = 0$ ✓ factors ✓ A(6 ; 0) (3)
10.3.1	$0 < x < 4$ or $0 \le x \le 4$ **OR/OF** $x \in (0\ ;\ 4)$ or $x \in [0\ ;\ 4]$	✓critical values ✓notation (2) **OR/OF** ✓critical values ✓notation (2)
10.3.2	$x > 2$ **OR/OF** $x \in (2\ ;\ \infty)$	✓ 2 ✓ notation (2) **OR/OF** ✓ 2 ✓ notation (2)
10.4	$f(x) = h(x-1) = -(x-1)^3 + 6(x-1)^2$ $f(0) = 7$ $7 < k < 32$ or $k \in (7\ ;\ 32)$	✓ $k < 32$ ✓ new y-intercept = 7 ✓ $7 < k < 32$ (3)
		[15]

QUESTION/VRAAG 11

11	Time = $\frac{20}{x}$ Cost = (water cost per hour × time) + (kms × R/km) $C(x)=1{,}6\times\left(\frac{20}{x}\right)+20\left(1{,}2+\frac{x}{4000}\right)$ $C(x)=\frac{32}{x}+24+\frac{x}{200}$ $C'(x)=-\frac{32}{x^2}+\frac{1}{200}=0$ $x^2=6\,400$ $x=80\,km/h$	✓ $\frac{20}{x}$ ✓ $1{,}6\times\left(\frac{20}{x}\right)$ ✓ $20\left(1{,}2+\frac{x}{4000}\right)$ ✓ $C(x)=\frac{32}{x}+24+\frac{x}{200}$ ✓ $C'(x)=-\frac{32}{x^2}+\frac{1}{200}$ ✓ $C'(x)=0$ ✓ answer (A) (7)
		[7]

QUESTION/VRAAG 12

12.1.1	No, because P(A and B) $\neq 0$	✓ answer and reason (1)
12.1.2(a)	$P(\text{A and B})=0{,}3 \qquad P(\text{only B})=0{,}2$ $P(\text{A and B})=P(\text{A})\times P(\text{B})$ $0{,}3=P(\text{A})\times 0{,}5$ $P(\text{A})=0{,}6$ $P(\text{only A})=0{,}3$	✓ $P(\text{A and B})=P(\text{A})\times P(\text{B})$ ✓ 0,5 ✓ $P(\text{A})=0{,}6$ ✓answer (4)
12.1.2(b)	 $P(\text{not A or not B})=0,2+0,2+0,3=0,7$ **OR/OF** $P(\text{not A or not B})=1-P(\text{A and B})=1-0,3=0,7$ **OR/OF** $P(A' \text{ or } B')=P(A')+P(B')-P(A' \text{ and } B')$ $=0,4+0,5-0,2=0,7$	✓method ✓answer (2) **OR/OF** ✓method ✓answer (2) **OR/OF** ✓method ✓answer (2)

<table>
<tr><td>12.2.1</td><td>P(novel) = $\frac{3}{12} = \frac{1}{4}$</td><td>✓answer
(1)</td></tr>
<tr><td>12.2.2</td><td>12! = 479 001 600</td><td>✓✓ answer
(2)</td></tr>
<tr><td>12.2.3</td><td>
<table>
<tr><td>5</td><td>3!</td><td>8!</td><td>4</td></tr>
<tr><td>(Poetry)</td><td>(Novels all together)</td><td>(Arrangements of rest of the books including the novels)</td><td>(Drama)</td></tr>
</table>
P(start with poetry, end with drama AND all novels together)
= $\frac{5 \times 3! \times 8! \times 4}{12!}$
= $\frac{1}{99}$
</td><td>✓ 5 × 4
✓ 3! = 6
✓8!

✓ $\frac{5 \times 3! \times 8! \times 4}{12!} = \frac{1}{99}$ (A)
(4)</td></tr>
<tr><td></td><td></td><td>[14]</td></tr>
</table>

TOTAL/TOTAAL: 150

Copyright reserved/*Kopiereg voorbehou*